

Axioms, Properties and Definitions of Real Numbers

Definitions

1. Property of a number system – a fact that is true regarding that system
2. Axiom – a property that forms the framework for the system. It does not require any proof. We assume that it is true.
3. Term – a combination of numbers and variables that are multiplied together.
4. Like terms – two or more terms that have the identical variables raised to the same power(s).
5. Coefficient – the number multiplying a variable in a term. If there is no written number, it is assumed to be 1.
6. Expression – a combination of terms added together
7. Equation – A combination of terms added together that contains an equal sign
8. Inequality – A combination of terms added together that contains an greater than, less than, greater than or equal to, less than or equal to.
9. Factor – a combination of numbers and variables that divides into a term evenly.
10. Common Factor – a combination of numbers and variables that is a factor of each in an expression

Properties and Axioms of Real Numbers

Full name	Symbolically	In words	Example
Commutative Axiom of Addition	$a + b = b + a$	The sum of two numbers is independent of their order	$3 + 6 = 6 + 3$
Commutative Axiom of Multiplication	$a \cdot b = b \cdot a$	The product of two numbers is independent of their order	$3 \cdot 6 = 6 \cdot 3$
Associative Axiom of Addition	$(a + b) + c = a + (b + c)$	The sum of a group of numbers is independent of how I group them.	$(5 + 3) + 2 = 5 + (3 + 2)$
Associative Axiom of Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	The product of a group of numbers is independent of how I group them	$(2 \cdot 4) \cdot 7 = 2 \cdot (4 \cdot 7)$
Distributive Axiom of Multiplication over Addition	$a(b + c) = ab + ac$	The product of a number and the sum of numbers is identical to the sum of the product of a number and each individual number	$6(4x + 3) = 6 \cdot 4x + 6 \cdot 3$
Distributive Property of Multiplication over Subtraction	$a(b - c) = ab - ac$	The product of a number and the difference of two numbers is identical to the difference of the product of a number and each individual number	$6(4x - 3) = 6 \cdot 4x - 6 \cdot 3$
Definition of Subtraction	$a - b = a + (-b)$	To subtract one number from another number, simply add the first number to the opposite of the second.	$4 - 6 = 4 + (-6)$
Definition of Division	$a \div b = a \cdot \frac{1}{b}, b \neq 0$	To divide one number by another number, simply multiply the first number by the reciprocal of the second.	$6 \div 3 = 6 \cdot \frac{1}{3}$

Additive Identity Axiom	$x + 0 = x$	If I add zero to any number, I get that number.	$4 + 0 = 4$
Multiplicative Identity Axiom	$x \cdot 1 = x$	If I multiply any number by 1, I get that number.	$5 \cdot 1 = 5$
Additive Inverse Axiom	$x + (-x) = 0$	If I add any number to its opposite, the result is 0	$14 + (-14) = 0$
Multiplicative Inverse Axiom	$x \cdot \frac{1}{x} = 1, x \neq 0$	If I multiply any nonzero number by its reciprocal the result is 1	$17 \cdot \frac{1}{17} = 1$
Multiplication Property of 0	$0 \cdot x = 0$	If I multiply any number by 0, the result is 0	$0 \cdot 5 = 0$
Multiplication Property of -1	$-1 \cdot x = -x$	If I multiply any number by (-1), the result is the opposite of the original number	$-1 \cdot 5 = -5$
Zero product	If $a \cdot b = 0$ then either $a = 0$ or $b = 0$ or both	If the product of two numbers is 0, then at least one of the numbers must equal 0.	
Properties of Equality			
Transitive Axiom of Equality	If $x = y$ and $y = z$ then $x = z$	If any number is equal to two other numbers, the other two numbers must also be equal to each other	If $3 + 4 = 5 + 2$ and $5 + 2 = 6 + 1$ then $3 + 4 = 6 + 1$
Symmetric Axiom of Equality	If $x = y$ then $y = x$	The two sides of an equation can be reversed without affecting the equation.	
Reflexive Axiom of Equality	$x = x$	A number always equals itself.	
Addition Property of Equality	If $a = b$ then $a + c = b + c$	A number can be added to both sides of an equation without affecting the equation.	If $a + 6 = 3$ Then, $a + 6 + (-6) = 3 + (-6)$
Multiplication Property of Equality	If $a = b$ then $a \cdot c = b \cdot c$	Both sides of an equation can be multiplied by the same number without affecting the equation.	If $12x = 6$ Then $\frac{12x}{12} = \frac{6}{12}$

Identity	True	An identity is an equation that is <i>true</i> for all real numbers (\mathbb{R})	$4 = 4$
Conditional Equation		An equation that is true for some values of the variable but not for all.	$x = 6$
	False	An equation that simplifies to a false statement has no solution $\{\emptyset\}$	$10 = 3$
Rational Number		A number that can be written as the ratio of two integers.	$4.\bar{6}, 5.763, -26$
Irrational Number		A number that cannot be written as the ratio of two integers.	$\sqrt{2}, \pi$
Closure Axiom under Addition	If $x \in \mathbb{R}$ and $y \in \mathbb{R}$ then $x + y \in \mathbb{R}$	If x and y are real numbers, their sum is a real number	
Closure Axiom under Multiplication	If $x \in \mathbb{R}$ and $y \in \mathbb{R}$ then $x \cdot y \in \mathbb{R}$	If x and y are real numbers, their product is a real number	
Closure	A set of numbers is closed under a given operation if there is just one answer and that answer is also in that set.		Subtraction over positive numbers is not closed because $4 - 6 = -2$ and -2 is not in the set of positive integers.